Progress on stochastic background search codes for LIGO

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Abstract. One of the types of signals for which the LIGO interferometric gravitational wave detectors will search is a stochastic background of gravitational radiation. We review the technique of searching for a background using the optimally-filtered cross-correlation statistic, and describe the state of plans to perform such cross-correlations between the two LIGO interferometers as well as between LIGO and other gravitational-wave detectors, in particular the preparation of software to perform such data analysis.

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1. Data Analysis Techniques

In this section we review the data analysis technique to be used to detect a stochastic background of gravitational radiation. More details can be found in [1, 2].

1.1. Definitions

First, we limit attention to backgrounds which are cosmological in origin and thus can be assumed to be isotropic, unpolarized, Gaussian, and stationary. Subject to these assumptions, the stochastic gravitational-wave (GW) background is completely described by its power spectrum. It is conventional to express this spectrum in terms of the GW contribution to the cosmological parameter $\Omega = \rho/\rho_{\rm crit}$:

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}}{d \ln f} = \frac{f}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}}{df} \ . \tag{1}$$

Note that $\Omega_{\rm GW}(f)$ has been constructed to be dimensionless, and represents the contribution to the overall $\Omega_{\rm GW}$ per *logarithmic* frequency interval. In particular, it is *not* equivalent to $d\Omega_{\rm GW}/df$. Note also that since the critical density $\rho_{\rm crit}$, which is used in the normalization of $\Omega_{\rm GW}(f)$, is proportional to the square of the Hubble constant H_0 [3], it is convenient to work with $h_{100}^2 \Omega_{\rm GW}(f)$, which is independent of the observationally determined value of $h_{100} = \frac{H_0}{100\,{\rm km/\,s/\,Mpc}}$.

1.2. Cross-Correlation

Since a stochastic signal is by definition random, it is impractical to look for one in the output of a single gravitational wave detector [4]. However, the effects of such a signal can be detected by cross-correlating the outputs of two independent detectors. If the output h_1 (h_2) of the first (second) detector consists of a term s_1 (s_2) due to stochastic gravitational waves and a term n_1 (n_2) due to instrument noise, and the noise in each instrument is assumed to be uncorrelated both with the GW signal and the noise in the other instrument, the only surviving term in a time averaged correlation $\langle h_1 h_2 \rangle$ is the term $\langle s_1 s_2 \rangle$ due to the gravitational wave background.

In practice, one defines a *cross-correlation statistic*

$$Y_Q = \int dt_1\, dt_2\, h_1(t_1)\, Q(t_1 - t_2) h_2(t_2) = \int df\, \widetilde{h}_1^*(f)\, \widetilde{Q}(f)\, \widetilde{h}_2(f) \tag{2}$$

which is weighted by a filter $\widetilde{Q}(f)$. In the presence of a stochastic gravitational wave signal, both the mean μ and variance σ^2 of the cross-correlation statistic will grow linearly with time, so the signal-to-noise ratio μ/σ will grow as the square root of the observation time.

Given a GW background spectrum and a pair of detectors, the signal-to-noise ratio is maximized by using the *optimal filter*

$$\widetilde{Q}(f) \propto \frac{f^{-3}\Omega_{\rm GW}(f)\gamma_{12}(f)}{P_1(f)P_2(f)} \ . \tag{3}$$

The denominator, containing the power spectral densities $P_{1,2}(f)$ of the noise in the two detectors, serves to suppress the contributions to the cross-correlation statistic from frequencies where one or both detectors are "noisy" and most correlations are therefore likely to be accidental. The numerator represents the average unweighted cross-correlation between the outputs of two detectors, and depends on both the

spectrum $\Omega_{\mathrm{GW}}(f)$ of the expected gravitational wave background and the locations and orientations of the two detectors. The latter is described by the *overlap reduction function* $\gamma_{12}(f)$ [5].

The overlap reduction function is equal to unity for the case of a pair of interferometers (IFOs) at the same location with their arms aligned, and is suppressed as the detectors are rotated out of alignment or separated from one another. The frequency dependence comes about for the following reason: if the wavelength of a wave is comparable to or smaller than the separation between two detectors, the detectors will see different phases of the wave at the same time, and this phase difference will depend on the direction of propagation of the wave. Since the stochastic GW background is assumed to be isotropic, averaging over different propagation directions suppresses the sensitivity of a pair of detectors to high-frequency waves. For example, a wave whose wavelength is twice the distance between the two detectors will drive them 180° out of phase if it travels along the line separating them, but *in* phase if its direction of propagation is perpendicular to this line. Figure 1 shows the overlap reduction functions for several detectors of interest.

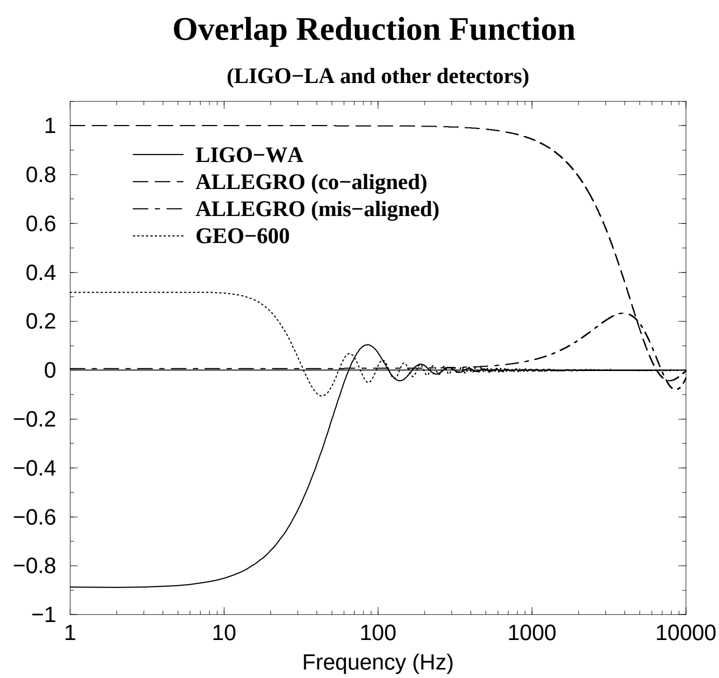

Figure 1. The overlap reduction function for several pairs of gravitational wave detectors. In each case, one of the detectors is the LIGO-LA site in Livingston, Louisiana. The curve labelled "LIGO-WA" shows the overlap with the site in Hanford, Washington; the one labelled "GEO-600" is for LIGO-LA and the GEO-600 site in Hannover, Germany, and the curves labelled "ALLEGRO" refer to the ALLEGRO resonant bar detector in Baton Rouge, Louisiana. The ALLEGRO experimental setup allows for the orientation of the detector to be changed. "ALLEGRO (co-aligned)" shows the overlap reduction function when ALLEGRO is oriented approximately parallel to one of the arms of LIGO-LA; "ALLEGRO (mis-aligned)" corresponds to an orientation 45° away from this.

1.3. Cross-Correlation Spectrum

In early data analysis applications, cross-correlated noise is likely to produce considerable spurious contributions to the integral (2). A useful diagnostic tool will thus be the *cross-correlation spectrum*

$$Y(f) = \widetilde{h}_1^*(f)\,\widetilde{Q}(f)\,\widetilde{h}_2(f) \ , \tag{4}$$

which is simply the integrand of (2); this may enable us to see directly the impact of these cross-correlated noise sources on the cross-correlation statistic.

2. Setting upper limits with LIGO

A two-week engineering run for the LIGO IFOs is planned for around the end of 2001, involving the kilometer-scale IFOs in Livingston, Louisiana [6] and Hanford, Washington [7]. The 600 meter GEO IFO [8] in Hannover, Germany and the ALLEGRO [9] resonant bar detector at Louisiana State University are also planning to operate during the same two-week period. Four groups have been formed to use the data taken from LIGO and the other instruments to set upper limits on various types of GW signals, including stochastic signals [10]. The stochastic group plans to set an upper limit on the strength of a stochastic background, assuming it has the form $\Omega_{\rm GW}(f) = $ constant, with the goal of improving on the existing best upper limit of $\Omega_{\rm GW}(f) \lesssim 60$ [11].

The roles to be played by various pairs of detectors in this effort are largely driven by their separation and alignment relative to one another, as quantified by the overlap reduction function (see Figure 1).

2.1. Correlations between LIGO-LA and LIGO-WA

The distance between the two LIGO sites is approximately 3000 km, which makes the light-travel time between the two sites about 10 ms. Thus the two sites are separated by half a wavelength for waves with a frequency of 50 Hz, and as shown in Figure 1 the overlap reduction function first crosses zero at a slightly higher frequency. So the overlap reduction function limits the sensitivity of this pair of detectors at high frequencies; below around 40 Hz, the seismic noise in the detectors will squash the optimal filter (3). The net effect (as illustrated in figure 21 of [2]) is that for LIGO initial design sensitivity, most of the support of the optimal filter lies between 50 and 250 Hz.

2.2. Correlations between LIGO-LA and ALLEGRO

The ALLEGRO bar detector is far closer to the LIGO Livingston site than the LIGO Hanford site is, with only about 40 km separating the two and a “half-wavelength” frequency of 3750 Hz. Thus the observing geometry allows for observations of correlations out to much higher frequencies. On the other hand, the sensitivity of ALLEGRO is concentrated in two narrow frequency bands in the vicinity of 900 Hz, so correlations between ALLEGRO and LIGO Livingston probe a different part of the frequency domain than correlations between the two LIGO detectors.¶

¶ Another consequence of the proximity between the two detectors is that there is likely to be a lot of cross-correlated noise; a method [12] has been proposed to account for this noise by measuring the cross-correlation for different alignments of the ALLEGRO bar.

2.3. Correlations between LIGO-LA and GEO-600

Since the GEO-600 detector is rather distant from the LIGO sites (over 7500 km from Livingston, corresponding to a "half-wavelength" frequency of only 20 Hz), the small overlap reduction function will render the GEO-600/LIGO-LA pair (the better of the two) considerably less sensitive to gravitational waves than the LIGO-LA/LIGO-WA pair. The primary interest in performing this correlation is thus the information it will provide about cross-correlated noise rather than a contribution to the upper limit on stochastic background strength.

3. Data Analysis Routines

3.1. Routines in the LIGO Numerical Algorithms Library (LAL)

The data analysis technique described in Section 1 will be implemented within the LIGO data analysis system (LDAS) [13] using C routines from the LIGO numerical Algorithms Library (LAL) [14]. We have written and tested LAL routines to perform the various parts of the analysis (calculating the overlap reduction function, constructing the optimal filter, etc.).

Care has been taken to make these routines general enough to be applied to both IFO data (e.g., from LIGO and GEO) and data from resonant bar detectors such as ALLEGRO. Two major issues have required some care in this regard:

First, the treatment of detector geometry (used in constructing the optimal filter) needed to be general enough to describe both interferometric and resonant detectors. This was accomplished by defining a data structure within LAL which described an idealized earthbound detector in terms of its location and tensor response to gravitational waves [15].

Second, since the sensitivity band of ALLEGRO is at a relatively high frequency compared to its bandwidth, its gravitational wave signal is heterodyned before being discretely sampled [16]. By multiplying the time-domain signal by a complex exponential oscillating at a base frequency, one effectively shifts the frequency band represented in the discrete signal (whose full width is equal to the sampling frequency) so that it is centered at the base frequency rather than at DC (0 Hz). To allow for this, the LAL routines had to be written to deal with complex as well as real time series.

3.2. Driver Routines in LALWrapper

The interface between the C++ LDAS environment and the LAL C library is known as LALWrapper [17]. LALWrapper contains a number of dynamically-linked shared objects which can be used to "drive" various search algorithms in LAL. We have written two LALWrapper shared objects: `libldasstochastic.so` calculates the cross-correlation spectrum (4) between two interferometric detectors, and `libldasstochasticbar.so` does the same for correlations between an IFO and a resonant bar detector. (Eventually, we plan to integrate the functionality into a single, generalized search engine.)

The general behavior of LALWrapper code is to execute parallel searches on one or more nodes of a Beowulf cluster [18], with each "slave" node reporting to the "master" node at least ten times. Both stochastic background search engines use the following

algorithm to calculate cross-correlation spectra for a sequence of short consecutive time intervals:

(i) Equal-length stretches of data from a pair of detectors are input, along with power spectra and response functions, and some search parameters.

(ii) The data streams are each divided into ten or more shorter-length segments.

(iii) An optimal filter is constructed using the auxiliary inputs and parameters describing the choice of detectors, etc.

(iv) In turn, each corresponding pair of data segments is Fourier-transformed and the cross-correlation spectrum calculated using the optimal filter.

Enhancements to be made for the scientific data runs which will begin in 2002 include: (i) Rather than constructing a single optimal filter based on the $\Omega_{\rm GW}(f) =$ constant model, we will choose a set of points in the parameter space of stochastic background models[19] and filter the data in parallel with a "grid" of optimal filters, one optimized according to (3) for each $\Omega_{\rm GW}(f)$ model [20]; (ii) To set an upper limit or make a measurement of the strength of the stochastic background, we will calculate the cross-correlation statistic for the whole data stream(s) by integrating the cross-correlation spectrum for a given segment over frequency and then adding the contributions from all the segments.

3.3. Mock Data Challenge

The shared objects `libldasstochastic.so` and `libldasstochasticbar.so` were among the data analysis routines tested at the Burst-Stochastic Mock Data Challenge[21], held at the Massachusetts Institute of Technology 4-10 September 2001. The shared object for IFO-IFO correlations was tested with trivial and non-trivial synthetic data and produced the expected results in each case; it was also used to analyze 15 minutes of data taken by the two LIGO IFOs during a recent engineering run, to verify that it could do so without failing. The IFO-bar shared object, being slightly less mature, was not tested as extensively. However, some of the tests which were run produced unexpected results which appear to be due to the use of single precision arithmetic in several LAL routines. The finer frequency resolution required by the sharp spectral features in the bar response function may require the use of double precision.

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References

[1] Allen B 1997 in *Proceedings of the Les Houches School on Astrophysical Sources of Gravitational Waves*, eds Marck J A and Lasota J P, Cambridge, 373
[2] Allen B and Romano J D 1999 *Phys. Rev.* **D59** 102001;
Allen B and Romano J D 1997 *Preprint* gr-qc/9710117
[3] Kolb E W and Turner M S 1990 *The Early Universe* (Addison-Wesley, 1990)
[4] Christensen N 1992 *Phys. Rev.* **D46** 5250
[5] Flanagan É É 1993 *Phys. Rev.* **D48** 2389
Flanagan É É 1993 *Preprint* astro-ph/9305029
[6] http://www.ligo-la.caltech.edu/
[7] http://www.ligo-wa.caltech.edu/
[8] http://www.geo600.uni-hannover.de/
[9] http://gravity.phys.lsu.edu/allegro/
[10] Allen B et al 2001 LIGO technical document LIGO-T010017-Z
http://feynman.utb.edu∼joe/research/stochastic/upperlimits/
[11] Astone P, Ferrari V, Maggiore M and Romano J D 2000 *Int. J. Mod. Phys.***D9** 361
[12] Finn L S and Lazzarini A *Preprint* gr-qc/0104040
[13] http://www.ldas-sw.ligo.caltech.edu/
[14] http://www.lsc-group.phys.uwm.edu/lal/
Allen B et al 1999 LIGO technical document LIGO-T990030-E
[15] LAL Software Documentation, version 0.9, Section 9.2; available from [14]
[16] Mauceli E et al 1996 *Phys. Rev.* **D54** 1264
Mauceli E et al 1996 *Preprint* gr-qc/9609058
[17] Blackburn K 1999 LIGO technical document LIGO-T990097-E
http://www.lsc-group.phys.uwm.edu/mpigroup/
[18] http://www.beowulf.org/
[19] Ungarelli C and Vecchio A 2001 in preparation
[20] Maggiore M 2000 *Phys. Rep.***331** 28
Maggiore 1999 *Preprint* gr-qc/9909001
[21] Authors TBD 2001 LIGO technical document number TBD entitled “Burst/Stochastic Mock Data Challenge”